Pg 492 (9.2) # 5-12 (~9), 24, 26

(5) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

$\sin 2x = (2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \dots (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$

$\forall x$

$= 2x - \frac{8x^3}{6} + \frac{32x^5}{120} - \dots$

$= 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 - \dots - (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$

(6) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + \dots$

$\ln(1-x) = -x - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} + \dots + (-1)^{n-1} \frac{(-x)^n}{n} + \dots$

$(-1, 1]$

$= -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots + (-1)^{2n-1} \frac{x^n}{n} + \dots$

(7) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^n \frac{x^{2n+1}}{2n+1}$

$\tan^{-1} x^2 = x^2 - \frac{(x^2)^3}{3} + \frac{(x^2)^5}{5} + \dots (-1)^n \frac{(x^2)^{2n+1}}{2n+1}$

$-1 \le x \le 1$

$= x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} + \dots (-1)^n \frac{x^{4n+2}}{2n+1}$

(8) $e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}$

$7xe^x = 7x\left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}\right)$

$\forall x$

$= 7x + 7x^2 + \frac{7x^3}{2!} + \dots \frac{7x^{n+1}}{n!}$

(10) $x^2 \cdot \cos x = x^2\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots + (-1)^n \frac{x^{2n}}{(2n)!}\right)$

$= x^2 - \frac{x^4}{2!} + \frac{x^6}{4!} \cdots + (-1)^n \frac{x^{2n+2}}{(2n)!}$ $\quad \forall x$

(11) $\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n = \sum_{n=0}^{\infty} x^n$

$\frac{1}{1-x^3} = 1 + x^3 + (x^3)^2 + (x^3)^3 + \cdots$

$= 1 + x^3 + x^6 + x^9 + \cdots = \sum_{n=0}^{\infty} x^{3n}$

$\frac{x}{1-x^3} = x + x^4 + x^7 + x^{10} + \cdots = \sum_{n=0}^{\infty} x^{3n+1}$ $\quad |x| < 1$

(12) $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$e^{-2x} = 1 + (-2x) + \frac{(-2x)^2}{2!} + \frac{(-2x)^3}{3!} + \cdots + \frac{(-2x)^n}{n!}$

$= 1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \cdots \frac{(-2)^n x^n}{n!}$ $\quad \forall x$

(24) $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots + \frac{x^n}{(n+1)!}$

a) $f'(x) = \frac{1}{2} + \frac{2x}{3 \cdot 2 \cdot 1} + \frac{3x^2}{4 \cdot 3 \cdot 2 \cdot 1} + \frac{4x^3}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} + \dots$

$= \frac{1}{2} + \frac{1}{3}x + \frac{1}{8}x^2 + \frac{1}{30}x^3 + \frac{1}{144}x^4 + \dots$

$f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{3} + \frac{1}{4}x + \frac{1}{10}x^2 + \frac{1}{36}x^3$

$f'''(x) = \frac{1}{4} + \frac{1}{5}x + \dots$

$f^4(x) = \frac{1}{5} + \frac{1}{6}x \dots$ $\rightarrow$ $f^{10}(x) = \frac{1}{11}$

b) $g(x) = x \cdot f(x)$

$= x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

c) $g(x) = e^x - 1$

$\Rightarrow$

(26) $f(t) = \frac{2}{1-t^2}$ $\quad G(x) = \int_0^x f(t)\,dt$

a) $\frac{1}{1-t} = 1 + t + t^2 + \dots$

$$\frac{2}{1-t^2} = 2(1 + t^2 + t^4 + t^6 + \dots + t^{2n}) = 2\sum_{n=0}^{\infty} t^{2n}$$

b) $G(x) = \int_0^x \frac{2}{1-t^2} = 2 \cdot \tan^{-1} x$

$$2 \cdot \tan^{-1} x = 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \right)$$

(from pink sheet)